Possible Connections between the Fermion Trajectories

ISMAIL A. SAKMAR*
University of California, Santa Barbara, California
(Received 24 February 1964)

Regge poles appearing in the odd and even continuations of the meson-baryon scattering amplitudes as a function of the energy are investigated from the point of view of relating different fermion trajectories. Many of the existing strange particle resonances with odd and even parity can be related whereas there are no pion-nucleon resonances with the proper quantum numbers which can be related within this scheme. The expected form of the trajectories is given in the complex J plane and also as $\mathrm{Re}\,\alpha\ (W)$ and $\mathrm{Re}\,\alpha\ (W^2)$, the last one corresponding to the Chew-Frautschi diagram. With the discovery of new resonances further possibilities arise for the behavior of the fermion Regge trajectories, one of which is suggested for the 1765-MeV resonance.

It is known that the partial wave amplitudes $f_{J-\frac{1}{2}}$ and $f_{J+\frac{1}{2}}$ describing meson-baryon scattering have kinematical singularities as functions of the square of the barycentric energy. To avoid these singularities it is customary to define new amplitudes $h_{J-\frac{1}{2}}$ and $h_{J+\frac{1}{2}}$ which are functions of the barycentric energy. These amplitudes satisfy MacDowell's reflection principle[1]:

$$h_{J-\frac{1}{2}}(W)=-h_{J+\frac{1}{2}}(-W)\,, \tag{1}$$

where W is the c.m. total energy.

To use this relation for complex values of J one defines in the usual manner even and odd continuations of the amplitudes and obtains:

$$h_{J-\frac{1}{2}}{}^{e,0}(W)=-h_{J+\frac{1}{2}}{}^{e,0}(-W)\,. \tag{2}$$

This relation shows that if the amplitude $h_{J-\frac{1}{2}}{}^{e,0}(W)$ has the pole $J=\alpha(W)$ in the complex J plane then the amplitude $h_{J+\frac{1}{2}}{}^{e,0}(W)$ has the pole $J=\alpha(-W)$. Calling the first pole $\alpha_1(W)$ and the second $\alpha_2(W)$ one has the relation

$$\alpha_1(W)=\alpha_2(-W)\,. \tag{3}$$

On the basis of real analyticity and the reflection principle more can be inferred about the poles of these amplitudes which differ only in space parity. The function $\alpha_1(W)$ has at an arbitrary point W in the complex W plane the same value as the function $\alpha_2(W)$ at the point $(-W)$. Thus in the J plane $\alpha_1(W)$ and $\alpha_2(W)$ trajectories coincide, the only difference being in the sign of the parameter W at the same points. Since $\alpha_2(-W)$ is just the reflection of $\alpha_2(W)$ both these functions have the same value at $W=0$. Consequently, $\alpha_1(W)$ also coincides with $\alpha_2(W)$ at $W=0$ as was first pointed out by Gribov.[2]

If W is pure imaginary the two operations "taking the complex conjugate" and "taking the negative" become the same so that MacDowell principle and the real analyticity can further be combined to show that the poles of the two amplitudes become complex conjugates of each other.

Thus, writing $W=ib$, where b is a real number, one has from

$$\alpha_1(ib)=\alpha_2(-ib)\quad\text{(reflection principle) and} \tag{4}$$

$$\alpha_2(-ib)=\alpha_2{}^*(ib)\quad\text{(real analyticity)} \tag{5}$$

$$\alpha_1(ib)=\alpha_2{}^*(ib)\,. \tag{6}$$

This was also first shown by Gribov. Thus, in addition to the right-hand cut in $W^2=u$ plane which starts at $u=(M+m)^2$, where M is the fermion mass and m is the meson mass, there is also a left-hand cut starting at $u=0$.

Now let us say one of the trajectories, say $\alpha_1(W)$, makes a particle or a resonance. If the real part of $\alpha_1(W)$ keeps on decreasing for $W<0$ this will mean that the real part of $\alpha_2(W)$ will decrease for $W>0$ and will not make a particle. On the other hand, if $\alpha_2(W)$ also makes a particle this will mean that the real part of $\alpha_1(W)$ has to increase again for $W<0$. Thus, in this case, $\alpha_1(W)$ will have a minimum. Even if $\alpha_2(W)$ does not make a particle a minimum is still possible. Moreover, α will have a left-hand cut not only in u but also in W.

From all these considerations one would expect the fermion Regge trajectory to have a shape more or less like the one shown in Fig. 1. The loop for $W<0$ is chosen in the lower half-plane because if one moves from the positive real W axis in W plane into the upper plane and approaches the positive imaginary axis the pole in J plane should move from the positive W loop towards the positive imaginary W loop. One should not expect to meet the negative W loop on this way. One also can see

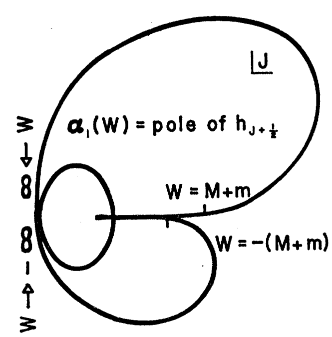

FIG. 1. A possible fermion Regge trajectory.

---

* Address as of 1 September 1964: University of Miami, Coral Gables, Florida.

[1] S. W. MacDowell, Phys. Rev. **116**, 774 (1960).

[2] V. N. Gribov, *Proceedings of the International Conference on High Energy Nuclear Physics, Geneva, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, 1962).

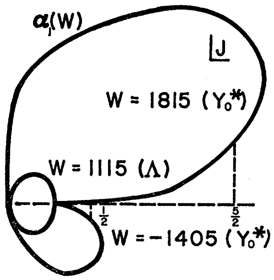

FIG. 2. Real part of the fermion Regge pole as function of the energy.

TABLE I. Strange particle resonances.

$I=0$	J	J parity	P	Mass (MeV)
	Λ $\frac{1}{2}$	$+$	$+$	1115
	Y_0^* $\frac{1}{2}$	$+$	$-$	1405
	Y_0^* $\frac{3}{2}$	$-$	$-$	1520
	Y_0^* $\geqslant\frac{5}{2}$	$+$	$+$	1815
$I=1$	Σ $\frac{1}{2}$	$+$	$+$	1190
	Y_1^* $\geqslant\frac{3}{2}$	$-$	$+$	1385
	Y_1^* $\frac{3}{2}$	$-$	$-$	1660
	Y_1^* $\frac{5}{2}$	$+$	$-$	1765

this by writing a dispersion relation for the function $\alpha(W)$ and looking at the relation derived from this by $W\rightarrow -W$ for $\alpha(-W)$. It will be seen that the imaginary part has the opposite sign. Here it should be remarked that the shown J-plane trajectories correspond to moving along the upper sides of the cuts in the W plane on both right and left. Thus on the left side in W one is following the complex conjugate of the physical amplitude.[3]

To summarize, we can say that the same trajectory (both loops included) can be considered as the Regge trajectory of the pole of $h_{J+\frac{1}{2}}$ or $h_{J-\frac{1}{2}}$. If it is taken to be $\alpha_1(W)$ the lower loop corresponds to negative energies and even if it reaches a physical J value it will not correspond to a physical particle. On the other hand, if we take the same trajectory to be $\alpha_2(W)$ which is the pole of $h_{J-\frac{1}{2}}$ then the lower loop will correspond to positive energies and if it makes a physical J value it will correspond to a physical particle. The upper loop now will correspond to negative energies and when it crosses physical J values this will not correspond to physical particles.

If we now plot $\mathrm{Re}\alpha(W)$ versus W, the curve corresponding to the trajectories of the Chew-Frautschi diagram[4] will probably look as shown in Fig. 2. $\mathrm{Re}\alpha_2(W)$ will be the reflection of this curve with respect to the $W=0$ axis. Thus, if $\mathrm{Re}\alpha_1(W)$ goes through a physical value in the negative energy region this will mean there is a particle with this spin, with the mass equal to the absolute value of this energy but with opposite parity; e.g., one has to invert the energy and the parity simultaneously.

From what was previously said, it follows that on the Chew-Frautschi diagram two fermion Regge trajectories with the same quantum numbers but opposite parities meet each other on the $u=0$ axis. Now, since

$$\frac{d\,\mathrm{Re}\alpha(u)}{du}=\frac{1}{2W}\frac{d\,\mathrm{Re}\alpha(W)}{dW} \tag{7}$$

if the minimum of $\mathrm{Re}\alpha(W)$ is not at $W=0$ the derivative $d\,\mathrm{Re}\alpha(u)/du$ will blow up at $u=0$. If the tangent to $\mathrm{Re}\alpha_1(W)$ at $W=0$ has the angle θ the tangent to $\mathrm{Re}\alpha_2(W)$ has the angle $(\pi-\theta)$ so that $d\,\mathrm{Re}\alpha(u)/du$ will go to the limit with different signs for each branch. That means that two trajectories with the same quantum numbers but different parities in Chew-Frautschi diagram not only meet each other on $u=0$ axis but are actually a single curve touching this axis. (Figure 3.) If the minimum is at $W=0$ then the right side of Eq. (7) is undetermined and we have to use l'Hôpital's rule:

$$\frac{d\,\mathrm{Re}\alpha(u)}{du}=\frac{d^2\,\mathrm{Re}\alpha(W)}{2dW^2}. \tag{8}$$

Since the second derivative of the two curves $\alpha_1(W)$ and $\alpha_2(W)$ which are reflections of each other in the $W=0$ axis and have a minimum at $W=0$ are equal, $d\,\mathrm{Re}\alpha(u)/du$ will be the same for both curves in the Chew-Frautschi diagram. But now instead of being tangent to the $u=0$ axis they will be tangent to some other line.

Table I gives a list of the fermions the Regge trajectories of which might be connected as suggested by our considerations. These Regge poles would appear in different $\bar{K}N$ scattering amplitudes.[5] Let us therefore discuss these amplitudes briefly:

1. $I=0$. *Zero Isotopic Spin Case*

(a) Odd continuation amplitudes.

(I) $h_{J+\frac{1}{2}}{}^0(W)$. For the odd continuation amplitude to be physical J has to be $\frac{3}{2}$, $\frac{7}{2}$, $\cdots$

spin$=\frac{3}{2}$, $\cdots$

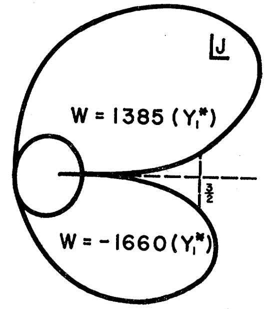

FIG. 3. Real part of the fermion Regge pole as function of the energy squared.

[3] G. F. Chew (private communication).

[4] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 8, 41 (1962).

[5] I. A. Sakmar, thesis, Lawrence Radiation Laboratory Report UCRL-10834 (May 1963) (unpublished).

orbital angular momentum $l=2$

$$\text{parity}=(-)\text{ if }\bar{K}\text{ parity is }(-)$$

$$J\text{ parity}=(-)^{J-\frac{1}{2}}=(-).$$

1520-MeV $Y_0{}^*$ has the quantum numbers of this channel.

(II) $h_{J-\frac{1}{2}}{}^0(W)$.

$$\text{Spin}=\tfrac{3}{2},\ \cdots$$

orbital angular momentum $l=1$

$$\text{parity}=+\text{if }\bar{K}\text{ parity is }(-)$$

$$J\text{ parity}=(-1)^{J-\frac{1}{2}}=(-).$$

There is no particle with these quantum numbers.

(b) Even continuation amplitudes.

(I) $h_{J+\frac{1}{2}}{}^0(W)$ is physical for $J=\frac{1}{2},\ \frac{5}{2},\ \cdots$. This gives us

$$\text{spin}=\tfrac{1}{2},\ \cdots$$

orbital angular momentum $l=1,\ \cdots$

$$\text{parity}=+\text{if }\bar{K}\text{ parity is }(-)$$

$$J\text{ parity}=(-1)^{J-\frac{1}{2}}=+.$$

A particle with these quantum numbers is Λ. Also the 1815-MeV $Y_0{}^*$ should be on the same trajectory if the assigned quantum numbers are correct.

(II) Now let us consider the other amplitude which is related to $h_{J+\frac{1}{2}}{}^e$ by MacDowell's reflection principle; e.g., $h_{J-\frac{1}{2}}{}^e(W)$. For this amplitude we have

$$\text{spin}=\tfrac{1}{2}$$

orbital angular momentum $l=0$

$$\text{parity}=(-)\text{ if }\bar{K}\text{ parity is }(-)$$

$$J\text{ parity}=(-1)^{J-\frac{1}{2}}=+.$$

1405-MeV $Y_0{}^*$ has these quantum numbers. If now the trajectories of these particles are the ones related by the reflection principle they will look as shown in Fig. 4. It is interesting to notice that the Regge pole has an imaginary part already at $W=M+m$.

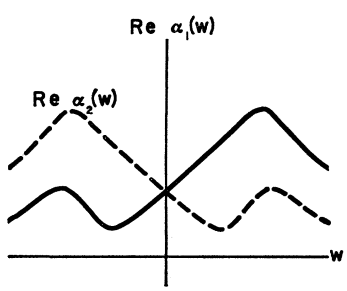

FIG. 4. Regge trajectory of the $I=0$ even continuation amplitudes if Λ_α and Λ_β are the trajectories related by MacDowell's reflection principle.

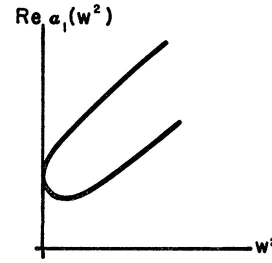

FIG. 5. Regge trajectory of the $I=1$ odd continuation amplitudes if Σ_δ and Σ_γ are the trajectories related by MacDowell's reflection principle.

2. $I=1$. Isotopic Spin 1 Case

(a) Odd continuation amplitudes.

(I) $h_{J+\frac{1}{2}}{}^0(W)$.

$$\text{Spin}=\tfrac{3}{2},\ \cdots$$

orbital angular momentum $l=2$

$$\text{parity}=(-)$$

$$J\text{ parity}=(-1)^{J-\frac{1}{2}}=(-).$$

1660-MeV $Y_1{}^*$ has probably these quantum numbers.

(II) $h_{J-\frac{1}{2}}{}^0(W)$.

$$\text{Spin}=\tfrac{3}{2},\ \cdots$$

orbital angular momentum $l=1$

$$\text{parity}=+$$

$$J\text{ parity}=(-1)^{J-\frac{1}{2}}=(-).$$

1385-MeV $Y_1{}^*$ has the quantum numbers of this channel. The possible trajectories of the Regge poles for $I=1$ in the odd continuation amplitudes are shown in Fig. 5.

(b) Even continuation amplitudes.

(I) $h_{J+\frac{1}{2}}{}^e(W)$.

$$\text{Spin}=\tfrac{1}{2},\ \cdots$$

orbital angular momentum $l=1,\ \cdots$

$$\text{parity}=+$$

$$J\text{ parity}=(-1)^{J-\frac{1}{2}}=+.$$

The particle with these quantum numbers is Σ.

(II) $h_{J-\frac{1}{2}}{}^e(W)$.

$$\text{Spin}=\tfrac{1}{2},\ \cdots$$

orbital angular momentum $l=0,\ \cdots$

$$\text{parity}=(-1)$$

$$J\text{ parity}=(-1)^{J-\frac{1}{2}}=+.$$

Recently, a resonance at 1765 MeV has been observed in Berkeley[6] which seems to have the quantum numbers of

[6] A. Barbaro-Galtieri, A. Hussain, and R. D. Tripp, Phys. Letters **6**, 296 (1963).

this channel. The state $D_{5/2}$ is suggested for this resonance. If this assignment is correct the question arises as to where the first recurrence of this trajectory with spin $\frac{1}{2}$ is. From the slopes of the other trajectories and from the fact that there is no particle between 1190 and 1765 MeV with the quantum numbers of this resonance one would expect its trajectory to lie higher than Σ trajectory. But also below 1190 MeV there is no particle with the quantum numbers of this resonance. Thus we are faced with the alternative that the odd parity trajectory crosses the spin $\frac{1}{2}$ line in the positive energy region thus having the wrong region for the odd trajectory and having the wrong slope for the even trajectory. A wrong slope would give a negative width and would not correspond to a particle. This possibility is shown in Fig. 6.

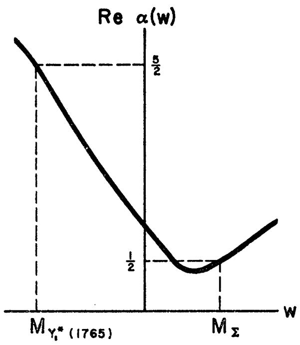

FIG. 6. A possible connection between the trajectories of Σ and 1765-MeV Y_1^*.

PHYSICAL REVIEW VOLUME 135, NUMBER 1B 13 JULY 1964

Dispersion Approach to Two-Body Weak Decays*

MASAO SUGAWARA
Department of Physics, Purdue University, Lafayette, Indiana

(Received 6 February 1964)

A dispersion theoretic approach to two-body weak decays is discussed in which the masses of the particles are regarded as constants. In this approach, an analyticity assumption is introduced for the invariant decay amplitudes which are defined off the energy-momentum shell. These amplitudes are invariant functions of three invariant variables. The singularities and the dispersion relations for these amplitudes are very similar to those assumed by Mandelstam in the case of scattering. As examples, the pionic decays of hyperons and the leptonic decays of pions and kaons are discussed in detail in the first order with respect to the weak Hamiltonian. It is assumed in the present approach that there exists a weak Hamiltonian which is localized in the sense of the present local field theory and can be treated as a small perturbation. In the case of the former example, it is shown that there are three kinds of pole terms corresponding to the above three invariant variables and that these pole terms are identical with those assumed in the pole approximation due to Feldman, Matthews, and Salam. The invariant decay amplitude in the case of the latter example becomes a constant in the present approach, if the electromagnetic correction is ignored. This is to be contrasted with various dispersion relations proposed in the conventional approach in which the mass of the pion is regarded as the variable. The dispersion theoretic version of the usual $V-A$ theory of the weak interaction is then constructed, in which the invariant decay amplitude (being a constant) is regarded essentially as the weak coupling constant. The experimental data concerning these leptonic decays indicate that the weak coupling constant defined this way is independent of not only whether the charged lepton is the electron or the μ meson, but also whether the decaying particle is the pion or the kaon.

I. INTRODUCTION

IN the dispersion theoretic approach to scattering, one usually assumes analyticity of invariant scattering amplitudes with respect to the invariant combinations of the particle four-momenta. These four-momenta are subject to the over-all energy-momentum conservation and all remain on the respective mass shells. One then finds two independent invariant variables. If one assumes analyticity with respect to both of these variables, one obtains double dispersion relations for the invariant scattering amplitudes. This was done first by Mandelstam.[1]

Suppose one applies the same consideration to decay of a particle with mass M into two particles with masses M' and m, respectively. It is straightforward to define invariant decay amplitudes. However, one finds no invariant variables, if all the particle four-momenta remain on the respective mass shells and satisfy the over-all energy-momentum conservation. To see this, let p, p', and q be the four-momenta[2] of the particles with masses M, M', and m, respectively. The conditions that these momenta are on the respective mass shells and satisfy the over-all energy-momentum conservation

* Work supported by the National Science Foundation.

[1] S. Mandelstam, Phys. Rev. **115**, 1741 (1959).

[2] Our notation of the four-momentum p is such that the space components are those of the three-momentum **p**, and the fourth component is ip_0, where p_0 is the relativistic energy of this particle.